# General One-Dimensional Model of a New Composite Ultrasonic Transducer

Igor Jovanović, Uglješa Jovanović and Dragan Mančić

*Abstract* - This paper presents an original general one-dimensional model of a new high-power composite ultrasonic transducer. Modelling of the composite transducer is performed using a one-dimensional analysis, which includes only thickness oscillation modes while radial oscillations are ignored. Unlike the most one-dimensional models that do not consider impact of the prestressing bolt or include only a part of it, the realized one-dimensional model includes all transducer components, as well as a central bolt with its head. Additionally, in order to prove the correctness of the transducer modelling process using the proposed model, a transducer model that ignores the central bolt impact will also be presented. Verification of the proposed one-dimensional models is performed by comparing the modelled dependencies of input electrical impedance vs. frequency with the experimental results.

*Keywords* - High-power ultrasound, Composite ultrasonic transducer, One-dimensional modeling.

## I. INTRODUCTION

At the very beginning of ultrasound development its application was in sonars. In nowadays, ultrasound is in applied on solid bodies, liquids and gases with the desired effect. The main applications of high-power ultrasound are ultrasonic cleaning and machining of materials. The high-power ultrasonic system consists of an electromechanical transducer and a power source operating in a predefined frequency range. Since ultrasonic energy is transferred to working environment through coupling elements, acoustic power emitted by the transducer depends on acoustic impedance of working environment and achieved adjustment. Design of efficient ultrasonic transducer includes a detailed analysis of mechanical and electrical transducer characteristics during different operating conditions. If the transducer emits ultrasonic waves to a complex acoustic load, the resonant frequency will change due to the change of boundary conditions on the transducer working surface [1].

The most widely used modelling approach for ultrasonic transducers, found in literature, is application of one-dimensional theory using equivalent electromechanical circuits. Electromechanical circuits use Mason's theory to model piezoelectric ceramics and symmetric T quadruples to model passive transducer elements [2], [3]. Therefore, the new composite transducer, analysed in this paper, is presented in the simplest form as a network with two electrical and two mechanical approaches. When model includes a bolt and various electrical connections of the transducer, then the number of electrical and mechanical approaches in the electromechanical equivalent circuit increases. Presence of the bolt becomes significant when determining resonant frequency of transducers with small axial dimensions. On the other hand, the bolt impact can be ignored in case of transducers with longer metal attachments. In literatures [4], [5], [6], [7] and [8], the bolt impact is not taken into the account since it is considered to be negligible, while in literature [8] the T networks of particular passive elements are even more simplified. In literatures [9] and [10], impact of the bolt part passing through ceramics is analysed in the form of parallel connection of the corresponding T quadruples. The bolt part passing through ceramics is presented along with the quadrupole impedances in the mechanical part of the equivalent electromechanical circuit in literatures [11] and [12], or directly via wave equations in literature [13].

In this paper, modelling of the realized composite transducer with new structure, which represents a special unidirectional composite ultrasonic transducer [14], is performed. Prestressing of the structure is achieved using the central bolt which is not in the contact with the central mass. Consequently, the central mass performs compressions and expansions in cycles, simultaneously with the changes of axial dimensions of the entire transducer due to mutually opposite polarization of piezoceramic rings in active blocks. Most of the problems related to impedance and frequency adjustment, as well as to mechanical load couplings, can be avoided by employing such composite transducer.

The paper also presents the general one-dimensional model of the high-power composite ultrasonic transducer with the new structure, which takes into the account all transducer components including a central bolt with the bolt head. The model is represented as a passive electromechanical equivalent circuit, the application of this equivalent circuit is based on the idea that propagation speed of ultrasonic waves is equivalent to electric current, while mechanical force is equivalent to electric voltage. In order to prove the correctness of the described transducer modelling process by the proposed general model, the paper also presents the one-dimensional model that does not include the central bolt impact.

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## II. ANALYTICAL ONE-DIMENSIONAL MODELLING OF HIGH-POWER ULTRASONIC COMPOSITE TRANSDUCER

The complete general one-dimensional model of the piezoelectric composite transducer that takes into the account the dimensions of the bolt and its head as well as the piezoelectric properties of the exciting ceramic is shown in Fig. 1. The model represented as the equivalent circuit is developed with the one-dimensional theory [2], [3] and it is modified based on the new structure of the composite transducer presented in literature [14]. In the proposed one-dimensional model, input electrical impedance depends on dimensions, material characteristics and resonant frequencies of the each transducer part included in the model.

Elements of the circuit shown in Fig. 1 corresponding to the isotropic metal transducer parts made of different materials are calculated as:

$$Z_{i1} = jZ_{ci} tg \frac{k_i l_i}{2}, \; Z_{i2} = \frac{-jZ_{ci}}{\sin(k_i l_i)} \quad (1)$$

wherein $Z_{ci}=\rho_i v_i P_i$ and $k_i=\omega/v_i$ (for i=1, 2, 3, ..., 6) are characteristic impedances and the corresponding wave numbers. $\rho_i$ are densities, $l_i$ and $P_i$ are lengths and surface areas of the cross-sections, $v_i$ are the velocities of longitudinal ultrasonic waves propagation through the corresponding elements.

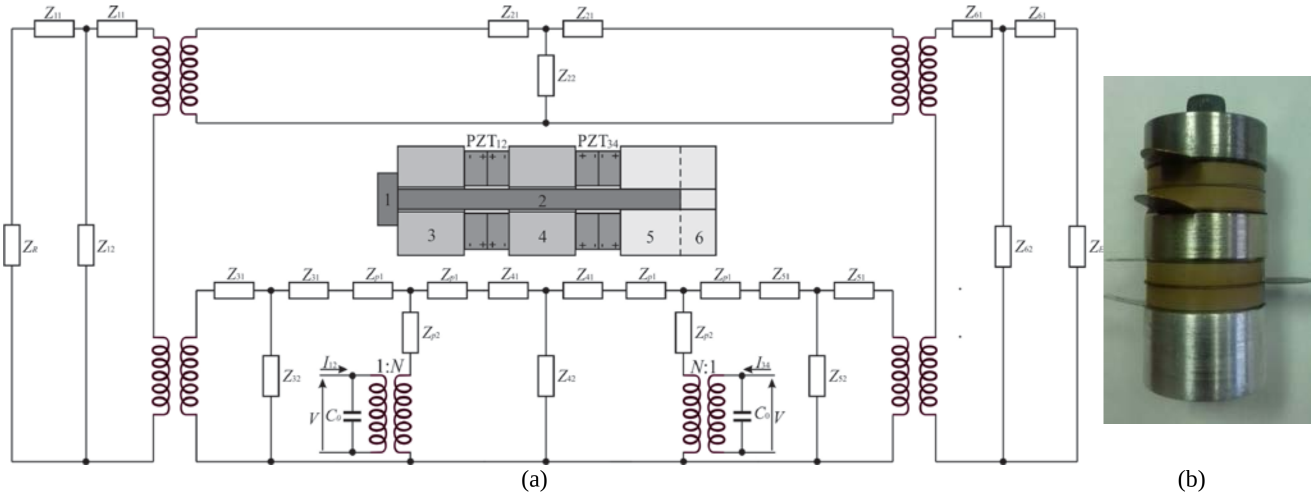
Elements of the circuit shown in Fig. 1 corresponding to the piezoceramic rings in the upper active layer ($PZT_{12}$) and the piezoceramic rings in the lower active layer ($PZT_{34}$) are determined as:

$$Z_{p1} = jZ_{cp} tg \frac{nk_p l_p}{2}, \; Z_{p2} = \frac{-jZ_{cp}}{\sin(nk_p l_p)} \quad (2)$$

wherein $Z_{cp}=\rho_p v_p P_p$ and $k_p=\omega/v_p$ are characteristic impedances and corresponding wave numbers, respectively. $\rho_p$, $l_p$, $P_p$ are densities, lenghts and surface areas of the piezoceramic cross-sections, $v_p$ are velocities of longitudinal ultrasonic waves propagation, respectively. The input electric voltages and currents are marked as $V$, $I_{12}$ and $I_{34}$. The piezoceramic models consist of capacitance $C_0=n\varepsilon_{33}^S P_p/l_p$, and ideal transformers with transmission ratios (electromechanical coefficients of the coupling) $N=h_{33}C_0/n$, wherein $n$ is the number of piezoceramic rings per the active layer (in the case of the particular composite transducer $n$ is 2). The piezoelectric properties of the transducer active layers are represented by piezoelectric constant $h_{33}$ and relative dielectric constant of the pressed ceramic $\varepsilon_{33}^S$.

Piezoceramic rings (between which is the central mass situated) are mechanically connected in series with emitter and reflector attachments. Emitter and reflector attachments are closed with acoustic impedances $Z_E$ and $Z_R$, which are in this case negligible because experimental measurements were conducted with unloaded transducers oscillating in the air. The metal bolt extends along the entire structure and is therefore connected mechanically in parallel to the remaining elements in the scheme. As already noted, most one-dimensional models do not include the bolt impact, or include only a part of it, hence the accuracy of the model in predicting the transducer behaviour is reduced, even in the case of transducers with longer metal attachments.

Due to the complexity of the expression for the composite transducer input impedance, observed on electrical approaches, as well as to prove the correctness of the transducer modelling by the proposed general one-dimensional model, which includes the impact of the prestressing bolt, the analytical model of the composite transducer, which ignores the central bolt impact is presented in the following text.



(a)

(b)

Fig. 1. (a) General one-dimensional model; (b) Realized composite transducer

If the central bolt impact is neglected, based on the equivalent circuit scheme shown in Fig. 1 ($Z_{11}=Z_{12}=Z_{21}=Z_{22}=0$) transducer input electrical impedance can be calculated as:

$$Z_e = \frac{Z_{12}Z_{34}}{N^2\left(Z_{12}+Z_{34}\right)+j2\omega C_0 Z_{12}Z_{34}} \tag{3}$$

wherein $\omega=2\pi f$ is angular frequency, $Z_{12}=V/I_{12}$ and $Z_{34}=V/I_{34}$ are input electrical impedances of the corresponding active layers obtained by following expressions:

$$Z_{12} = \frac{Z_{e5}-\frac{Z_{e2}}{Z_{e3}}Z_{e6}}{Z_{e4}-\frac{Z_{e1}}{Z_{e3}}Z_{e6}},\quad Z_{34} = \frac{Z_{e3}-\frac{Z_{e6}}{Z_{e5}}Z_{e2}}{Z_{e1}-\frac{Z_{e4}}{Z_{e5}}Z_{e2}} \tag{4}$$

In Eq. 4 the newly introduced equivalent impedances are obtained by following expressions:

$$\begin{gathered}
Z_{e1} = 1+\frac{Z_{p1}+Z_{41}+Z_{42}}{Z_{e7}}+\frac{Z_{42}}{Z_{e8}}, \\
Z_{e2} = Z_{42}+Z_{p2}+Z_{p1}+Z_{41}+\frac{Z_{p2}\left(Z_{42}+Z_{p1}+Z_{41}\right)}{Z_{e7}} \\
Z_{e3} = Z_{42}+\frac{Z_{p2}Z_{42}}{Z_{e8}}, \\
Z_{e4} = 1+\frac{Z_{p1}+Z_{41}+Z_{42}}{Z_{e8}}+\frac{Z_{42}}{Z_{e7}}, \\
Z_{e5} = Z_{42}+\frac{Z_{p2}Z_{42}}{Z_{e7}}, \\
Z_{e6} = Z_{42}+Z_{p2}+Z_{p1}+Z_{41}+\frac{Z_{p2}\left(Z_{42}+Z_{p1}+Z_{41}\right)}{Z_{e8}}
\end{gathered} \tag{5}$$

that is:

$$\begin{gathered}
Z_{e7} = \frac{\left(Z_{rf}+Z_{31}\right)Z_{32}}{Z_{rf}+Z_{31}+Z_{32}}+Z_{31}+Z_{p1}, \\
Z_{e8} = \frac{\left(Z_{ef}+Z_{(5+6)1}\right)Z_{(5+6)2}}{Z_{ef}+Z_{(5+6)1}+Z_{(5+6)2}}+Z_{(5+6)1}+Z_{p1}
\end{gathered} \tag{6}$$

Based on Eq. (3), expressions for resonant frequencies can be derived as:

$$Z_{12}Z_{34} = 0 \tag{7}$$

Expressions for antiresonant frequencies can be derived as:

$$N^2\left(Z_{12}+Z_{34}\right)+j2\omega C_0 Z_{12}Z_{34} = 0 \tag{8}$$

Based on Eqs. (7) and (8) it is obvious that the transducer frequency response depends on the material characteristics of its constituting parts and their geometric dimensions.

In the proposed transducer models, it is assumed that the circuit elements are ideal, i.e. they do not have losses. Losses can be included if piezoelectric constants and constants of elasticity of the transducer metal parts are in the form of complex numbers, in which the imaginary parts represent losses. These models allow only the thickness resonant modes to be predicted and, therefore, do not take into the account the inevitable radial resonant modes.

## III. SIMULATION AND EXPERIMENTAL RESULTS OF THE COMPOSITE TRANSDUCERS

By connecting acoustic impedances at the outer surfaces, the input electrical impedance, $Z_{ul}=U/I$ can be easily obtained. In order to compare results obtained by the models with experimental measurements, the modulus of transducer input impedance was determined, wherein due to the large range of impedance changes the decay function in the decibels was analyzed ($z_{ul}=20\log|Z_{ul}[\Omega]|$ [dB]). It is assumed that the surrounding environment is only air.

Table 1 shows dimensions of the individual transducer with following dimensions of the exciting piezoceramic rings Ø38/Ø13/6.35 mm, which are made of PZT4 piezoceramic equivalent material [15]. The emitter is made of dural while the reflector and the central mass are made of steel with the standard material properties.

TABLE I
DIMENSIONS OF COMPOSITE TRANSDUCER USED IN EXPERIMENTAL ANALYSIS

| Dimension [mm] | Composite transducer |
|---|---|
| $2L_1$ | 8 |
| $2L_2$ | 58 |
| $2L_3=2L_4$ | 11 |
| $2L_5$ | 16 |
| $2L_6$ | 21 |
| $2a_1$ | 13 |
| $2a_2$ | 8 |
| $2a_3=2a_4=2a_5=2a_6$ | 40 |
| $2b_3=2b_4$ | 9 |
| $2b_5=2b_6$ | 8 |

Impedance/gain-phase analyzer HP4194A was used to record the experimental characteristic of input electrical impedance vs. the frequency dependency which was compared to the analogue characteristic obtained by the proposed models.

Verification of the proposed one-dimensional models was performed by comparing the modeled characteristics of input electrical impedance vs. frequency dependency with the experimental measurements for the realized unloaded ultrasonic composite transducer. Modeled dependences were obtained firstly by using the proposed one-dimensional model that does not take into the account the central bolt impact (Eq. 3), and then by applying the proposed general one-dimensional model that takes into the account all transducer parts including the central bolt with its head (see Fig. 1).

Fig. 2 shows experimental and modeled dependencies of transducer input electrical impedance in case of the transducer with dimensions given in Table 1. As it can be seen from Fig. 2, there is a great similarity between modeled and experimental dependencies. Both one-dimensional models have satisfactory results when analyzing the transducer in the case of the basic resonant mode. Measured resonant frequency of the first resonant mode is 25.62 kHz. Resonant frequency obtained by the general one-dimensional model is 25.7 kHz and the error it makes in determining this resonant frequency is 0.31%. Resonant frequency obtained by the one-dimensional model that does not take into the account the bolt impact is 25.8 kHz and the error is 0.7%. The proposed model can predict the general shape of the fourth resonant mode (with the measured resonant frequency of 52.23 kHz) but with large deviations (the detailed analysis of determination of the higher resonant modes nature is presented in literature [14]).

In the presented case from the practical aspect, when designing a transducer with a one-dimensional method, a small difference between the precision of prediction of the resonant frequencies of the basic mode is negligible. It is more important to predict all the thickness modes with the appropriate model, and especially the modes close to the operating mode that can imact the transducer behavior, which was in this case achieved by the proposed general one-dimensional model.

Based on the comparison between two characteristics modeled with the one-dimensional theory, it can be assumed that the second resonant mode, obtained by the proposed general one-dimensional model, originates from the bolt impact.

As already stated, it is possible to model the thickness resonant modes using the one-dimensional theory. In this case, the modeled resonant frequencies of the first and the second resonant modes are greater than the ones measured, while for the fourth resonant (third thickness) mode they are lower than the measured resonant frequencies. This is probably due to the presence of the modes, which are not included by the model, that is, primarily radial resonant modes and other possible types of vibration modes.

Resonant modes depend on the coupling between multiple individual modes. A detailed analysis of the individual impact of the transducer parts on the resonant modes is given in literature [14], and it was carried out with the approximatively three-dimensional matrix model [16], which predicts the thickness and the radial oscillation modes, as well as their mutual couplings. In this case, the third resonant mode, which occurs at 44.21 kHz, is a radial resonant mode in conjunction with the second and fourth resonant modes. Impact on the third resonant mode frequencies have only radial changes in the characteristics of the emitter and piezoceramic rings in the lower active layer ($PZT_{34}$). Coupling of the third resonant mode with the second mode arises from the fact that radial changes of the piezoceramic rings characteristics in the lower active layer ($PZT_{34}$) also have impact on the second mode. Coupling of the third resonant mode with the fourth mode was created based on the impact of changes of the emitter characteristics in the radial direction. In addition, the greatest impact on frequencies of the fourth resonant mode have changes in characteristics of the central mass, the emitter and all piezoceramic rings in the axial direction.

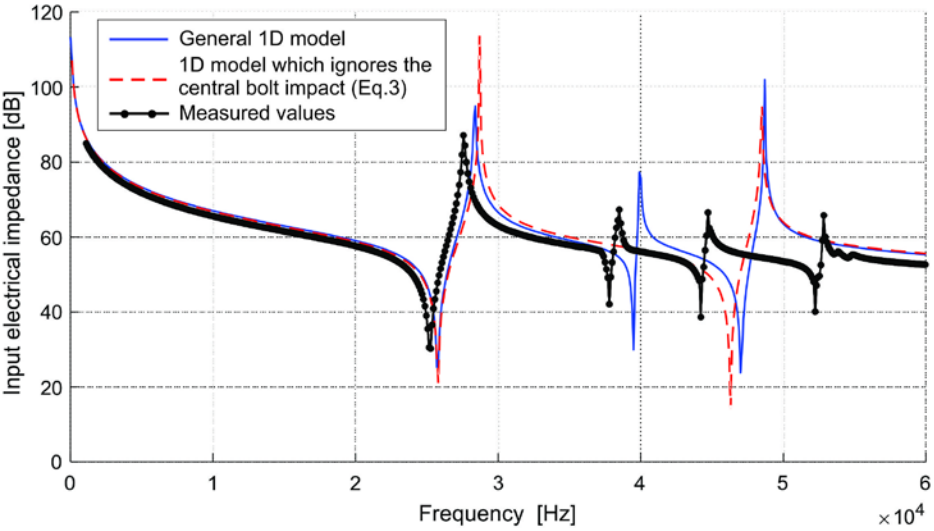


Fig. 2. Input electrical impedance vs. frequency for the realized composite transducer

Due to the strong coupling between these thickness modes with the radial mode, the proposed one-dimensional theory cannot make accurate prediction of the aforementioned resonant modes. Since the model does not take into the account mechanical and dielectric losses, which can be easily added using the characteristics of materials in the form of complex numbers, the minimal and the maximal impedance values are more pronounced on modelled dependencies compared to the measured dependencie.

## IV. CONCLUSION

To sum up the above analysis, the following conclusions may be drawn:

- Because the structure of the composite transducer is complex, its vibrational modes are more complex compared to the traditional sandwich longitudinal transducer. Therefore, more vibrational modes were created, and the frequency characteristics becomes complex.
- Presence of the resonant modes not included by the one-dimensional models is best seen from the dependency shown in Fig. 2. The third measured resonant mode does not represent a thickness resonance, hence the modelled dependencies of the third thickness mode are at lower frequencies than the measured dependencies. Similarly, the dependency of the second thickness mode modelled with the general one-dimensional model is at higher frequencies than the measured dependency.
- The theoretical analysis of the presented research is one-dimensional. It requires the radial dimension to be much lower than its longitudinal dimension. When the radial transducer dimension increases, the radial vibration becomes intense. In this case, the three-dimensional coupled vibration should be considered. In addition, with transducers with small axial dimensions compared to radial dimensions, presence of the bolt becomes important in determining the resonant frequency.

## ACKNOWLEDGEMENT

This work was supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia under the project TR33035.

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