Response Distribution Analysis for Oscillator Circuit Due to Process Variation

Miljan Petrović and Miljana Milić

Abstract - In this paper we will try to analyse the influence of different statistical shapes and features of the technology parameters distributions to component behaviour, and consequently to the circuit response distribution. In this way it will be possible to create the methodology that will widen the use of Monte Carlo analysis and make it applicable not only to process variation research, but also to modelling effects of IC component aging, yield estimation, etc. The procedure will be demonstrated on the simple oscillator circuit through accessing process variation features of 555 timer component, and is based on multiple LTspice simulations, and statistical tools of the Matlab programme.

Keywords – MC analysis, Process variations, Tolerance, Response distribution, PDF.

I. Introduction

The process variation is one of the main factors examined in PVT (Process Voltage Temperature) design of analog circuits, that deals with numerous naturally occurring variations. Particularly, process variation represents changes of the semiconductors' attributes during IC fabrication process. It can cause significant and affective distortion in the output performance of analog circuits due to characteristics mismatch. These circuit response variations can be efficiently predicted in order to avoid the misspecification of a particular circuit or device, reducing the overall yield.

The device mismatch can be defined as a small random variation in parameters of identically designed devices, which occurs during the IC manufacturing process [1]. The basic approach used by designers is increasing the size of devices sensitive to mismatch. This decreases relative error in the desired characteristic minimizing the mismatch. However, smaller devices and complex circuits require large scale Monte Carlo (MC) simulations in order to investigate the way individual mismatches affect the circuit in whole. Hence, it is of great importance to correctly set up and perform simulations.

In the field of process variation, two directions of research have been emerged. Exploring the effects of process variation is often necessary for studying the influence of age to component performance [2], [3], [4]. Second, and equally important application of the analysis result is in the circuit design process, where designers have to build a circuit which is insensitive to parameters' fluctuations due to process variation [5], [6], [7], [1]. Special attention should be paid to the research described in [7], where the use of MC analysis is avoided, which is rare in literature and opposite to other references dealing with this problem.

Statistical Analysis implies running tens to thousands of simulations so that designer can analyze the behaviour of the circuit in accordance to variations of the active components' manufacturing process. Process variation data for a certain component are usually provided by the manufacturer and obtained through systematic testing and measurements. The results of such measurements are being mapped to corresponding model files. However, this procedure can be far more consuming in time and resources for complex IC components. One of the goals of this paper is to stress out the possibility and efficiency of using a simulation based method, rather than the mentioned procedure, for accessing process variation features of the IC.

The aim of this study was to examine process variation features of a complex semiconductor component encompassed in an analog circuit. By performing detailed statistical analysis of the simulation results, effects of underlying distributions of component parameters on the circuit response are discussed. Also, an elaboration is given on the selection of measures of distribution of component parameters, in order to select the best fit for the particular purpose (process improvements, aging effects, testing of parametric defects, tolerance design, yield estimation, etc) [8].

In the following sections a demonstration of MC based statistical response distribution analysis shall be provided. A short description of the simple oscillator circuit and reasons for choosing it as a subject of the analysis method will be given. Next, the methodology of the statistical analysis, based on Spice simulations and statistical Matlab [9] tools, will be explained in more detail. As the result, we have demonstrated the analysis for the response distribution of a simple oscillator circuit and gave the explanation and conclusions of the obtained statistical measures. Further research will be listed in the conclusions.

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II. The Oscillator Circuit

Response distribution analysis is performed on an oscillator circuit shown in Fig. 1. The element whose process variation data are being examined is the NE555 timer IC. The circuit is described in LTspice [10]. The subcircuit for the IC is the original Linear Technology copyrighted netlist NE555, with additional changes as explained in Section 3.

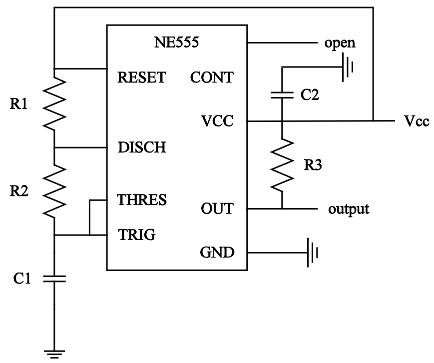

Fig. 1. NE555 based oscillator circuit

TABLE I
Values of elements in oscillator circuit

Circuit element	Value
R_1	5 kΩ
R_2	3 kΩ
R_3	1 kΩ
C_1	0.15 μF
C_2	0.01 μF
V_{CC}	15 V

The oscillator is an astable multivibrator generating rectangular periodic output signal. This circuit appears suitable for response analysis, since it generates a single scalar output value, which, in fact, is the fundamental frequency of the output signal; the waveform itself is not relevant to the analysis. Since the behaviour of the circuit relies on charging the capacitor C_1 through resistors R_1 and R_2, and discharging through R_2, these are the elements that define the output frequency as in Eq. (1):

$$f \approx \frac{1.44}{(R_1 + 2R_2)C_1} \tag{1}$$

where f is the working frequency, and resistances R_1 and R_2 and capacitance C_1 are values of circuit elements according to Fig. 1. Values of all the elements for the designed oscillator are listed in Table I. The corresponding frequency should be 872.7273 Hz.

However, after Spice simulation was performed, it was observed that due to specific component modelling the working frequency is 928.79 Hz. This value is taken as nominal in regard of the latter statistical analysis.

III. Methodology of Statistical Analysis

The first step in analysis of the described circuit is performing MC simulations. Several groups of data are acquired. Namely, the output signal of the circuit is generated in LTspice for 1000 simulations per each set of parameters. These parameters are: the tolerance T of passive elements of the NE555 Spice model, and the type of the statistical distribution, from which the values of these elements are generated. Since the most common types of distributions used in MC analysis are uniform [5], [6] and Gaussian [1], they both are implemented for the purpose of this paper. In the case of Gaussian distribution, the tolerance T implies that the variance of the distribution is $\sigma^2=(T/100)^2$, where T is expressed in percents. The mean of the distribution is the nominal value of the Spice element which is the part of the original netlist. In the case of the uniform distribution, T is in fact the tolerance, i.e. maximal relative distance between the generated value and the center of the distribution. The circuit displayed in Fig. 1 is implemented in LTspice with commands for transient MC simulations. Formulae for random number generation according to the described distributions are also included in the Spice netlist. For different types of distributions, separate netlists are extracted. Namely, the generation of passive elements' values for timer model is defined by the type of underlying distribution. On the other hand, values of external elements of the oscillator circuit are fixed. These include resistances R_1, R_2 and R_3, capacitances C_1 and C_2 and DC supply voltage V_{CC}.

Finally, a thousand output signals with the duration of 50ms (approximately 46.5 periods of oscillation) are generated for two types of distributions and and three particular tolerance values: T=2%, T=5% and T=10%. Consequently, these results are imported in Matlab for further statistical processing. A script has been written in order to calculate the fundamental frequency and transform sets of output signals into sets of frequency values. The frequency is derived by calculating the discrete Fourier transform (using FFT algorithm) and finding the location of the maximal amplitude component in the spectrum. In order to achieve good accuracy, some additional HP filtering was implemented.

After all the frequencies are acquired, significant statistical quantities are calculated for each of 6 sets. Namely, several central moments are estimated, which, in compliance with [11], can be defined as in (2):

$$\mu_n = E[(X - E[X])^n] \tag{2}$$

where μ_n stands for n^{th} central moment, X denotes the random variable, E is the expectation operator ($E[X]$ is the mean, i.e. the first moment of X), whereas n can have values 2, 3 and 4, respectively. Sample estimation is performed for the lowest four central moments, i.e. mean, variance, skewness and kurtosis. Used sample estimators are unbiased.

Empirical PDFs (probability density functions) were calculated next (histograms and kernel-smoothing density estimated functions), and compared to known mathematical distributions. Different information criteria was used to fit numerous distributions to generated data, and each gave the best fit with its minimal value. The used criteria are: Bayesian Information Criterion (BIC), Akaike Information Criterion (AIC), Akaike Information Criterion with correction for finite sample sizes (AICc), and negative log likelihood (NLogL). The implemented algorithm results in different best fits for cases of Gaussian and uniform element values distribution.

IV. Analysis Results

Tables II and III contain unbiased estimates of the four lowest central moments of the response distribution. Table II considers the Gaussian random generation, whereas Table III considers the uniform. The simulation of the nominal NE555 model gives the frequency of 928.79Hz. However, results from Table III show the relative error of the frequency mean is around 2% for all tolerances (the estimated mean is between 910Hz and 911Hz). It can be noticed that the estimated mean increases with the tolerance when Gaussian generation is used. According to the chosen underlying distribution for passive elements, it is clear that in general, means and variance estimates are greater in Table II than in Table III. It can be concluded that Gaussian MC analysis represent the worse one in comparison to the uniform MC.

TABLE II

Estimation of Moments of Response Distribution under Gaussian Distribution for Passive Elements Generation

tolerance / moment order	T=2%	T=5%	T=10%
1 (mean)	911.272387	913.097827	920.208811
2 (variance)	610.489362	3643.53864	15206.2522
3 (skewness)	0.04797167	0.17813189	0.46685908
4 (kurtosis)	3.04642211	2.88281022	3.07880775

TABLE III

Estimation of Moments of Response Distribution under Uniform Distribution for Passive Elements Generation

tolerance / moment order	T=2%	T=5%	T=10%
1 (mean)	910.936195	910.633842	910.840261
2 (variance)	134.645217	587.226402	2269.23186
3 (skewness)	-0.2468981	0.06193966	0.11390945
4 (kurtosis)	3.14433509	2.57091046	2.46468484

Data in Tables II and III correspond to Figs. 2 and 3 where kernel-smoothing PDF estimates of each set of frequencies is plotted. Conclusions derived from results of mean and variance are illustrated in these plots. Also, skewness and kurtosis are sometimes easier to examine visually. Namely, uniform based data show response distributions oriented more to the left, and Gaussian base to the right, which is confirmed both with plots and skewness estimates. However, skewness analysis only makes sense for unimodal distributions. It is also interesting that at lower tolerances T, response distributions tend to be multimodal and their PDFs show several local extremals, which is observable in Figs. 2 and 3.

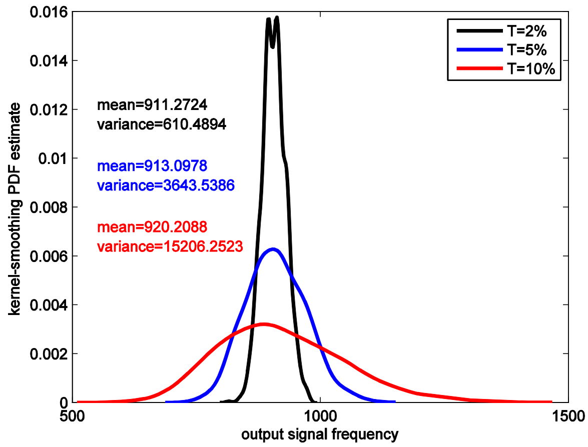

Fig. 2. Kernel-smoothing PDF estimate for tolerance values of 2%, 5% and 10% in the case of Gaussian distribution based generation of passive model elements

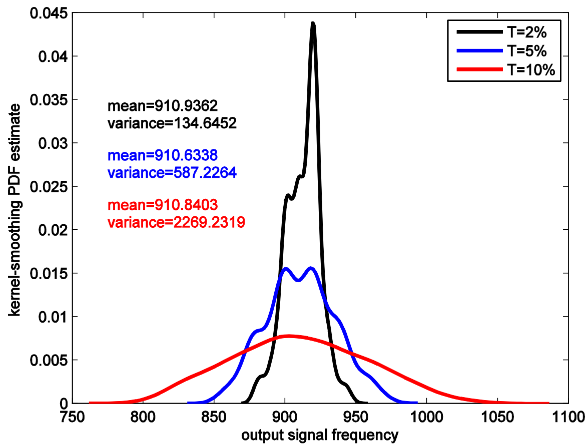

Fig. 3. Kernel-smoothing PDF estimate for tolerance values of 2%, 5% and 10% in the case of uniform distribution based generation of passive model elements

After estimation of central moments, the best fit for response distributions should be determined next. For each set of frequencies information criteria are calculated in order to check if the data comes from certain distributions. The investigated distributions are: generalized extreme value, Birnbaum-Saunders, inverse Gaussian, log-normal, gamma, Nakagami, Rician, normal (Gaussian), t location-scale, log-logistic, logistic, Weibull, extreme value, Rayleigh and exponential distributions [11], [12]. The calculated criteria for sorting the fits were: NLogL, AIC, AICc and BIC. Examples of fitting can be seen in Figs. 4 and 5. Fig. 4 shows normalized histogram (empirical PDF) and two best continuous PDF fits (generalized extreme value and inverse Gaussian), according to AIC, in the case of Gaussian generation and *T*=10%. Fig. 5 shows normalized histogram (empirical PDF) and two best continuous PDF fits (generalized extreme value and Birnbaum-Saunders), according to AIC, in the case of uniform generation and *T*=10%. Plotting more fits at the same time makes the figure unclear since PDFs differ slightly.

The results of the best fits for each set of data and each criterion are listed in Table IV. The letter G stands for the Gaussian based generation of passive elements, whereas the letter U stands for uniform one. Abbreviations are used for distributions names so that the table can be more legible. These are: Γ for gamma, GEV for generalized extreme value, BS for Birnbaum-Saunders, IG for inverse Gaussian, TLOC for t location-scale and R for Rician.

TABLE IV
BEST FIT RESPONSE DISTRIBUTIONS FOR EACH SET OF FREQUENCIES AND EACH CRITERION

criterion / dataset	NLogL	AIC	AICc	BIC
T=2%, G	Γ	Γ	Γ	Γ
T=5%, G	GEV	BS	BS	BS
T=10%, G	GEV	GEV	GEV	IG
T=2%, U	TLOC	R	R	R
T=5%, U	GEV	GEV	GEV	GEV
T=10%, U	GEV	GEV	GEV	GEV

Comparing the results for AIC and AICc leads to an observation that all the fits for these two criteria are the same. This confirms the fact that the sample size of 1000 taken in provided analysis in this paper is large enough to derive statistical conclusions.

Also, it is of great importance to look at results of AIC and BIC. Namely, these criteria are essentially different. AIC is asymptotically optimal in terms of average square error, whereas BIC has the consistency property (it converges to the true model for larger sample sizes) [13]. However, these differences can be negligible considering presented results. The only case in which the type of best fit distribution is different according to AIC and BIC is the case of *T*=10%, G. The compliance of AIC and BIC is particularly emphasized in the case of uniform based generation and lower tolerances.

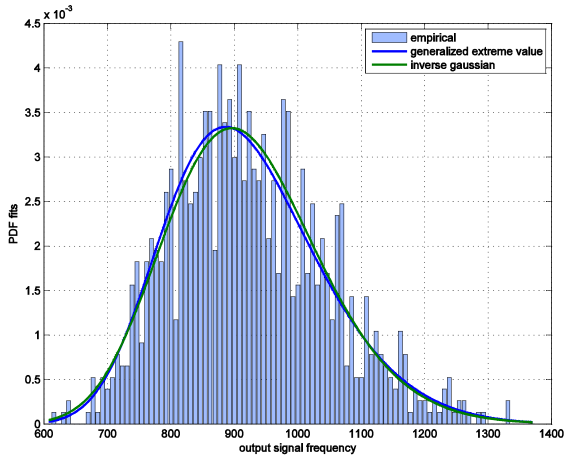

Fig. 4. Normalized histogram and two best fits for response distribution according to AIC at *T*=10% in the case of Gaussian distribution based generation of passive model elements

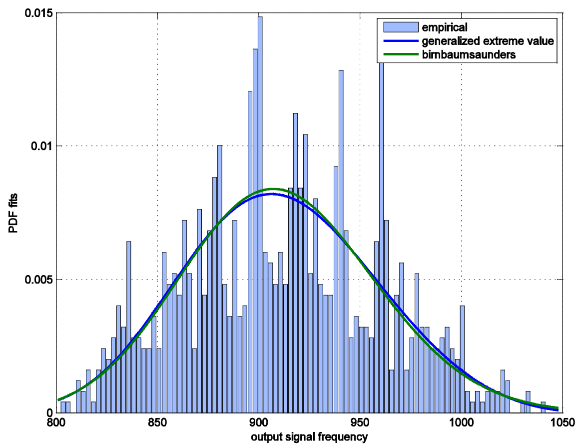

Fig. 5. Normalized histogram and two best fits for response distribution according to AIC at *T*=10% in the case of uniform distribution based generation of passive model elements

The best fits depend greatly on the value of tolerance *T*, especially in the case of Gaussian generation. However, generalized extreme value distribution is the most common best fit considering all cases. That is why GEV can be a reasonable assumption for response distribution under process variation of NE555, e.g. used in component aging analysis as in [2] and [4].

V. CONCLUSION

In this paper a methodology of MC based statistical analysis of an oscillator circuit has been described. Several significant conclusions were inferred through comprehensive examination of results. Those are referred to accessing process variation features of ICs. First, it was shown that, due to mapping of fabrication parameters to elements of the IC Spice model, effects of component's process variation can be inspected by random generation of passive elements of the model during MC simulations and analysis of the response distribution. Further work can be focused on similar analysis including generation of other Spice model parameters beside passive elements' values. In addition, this paper provides an elaboration on the statistical process of generation in the sense of determining the proper type of distribution.

Agreement between results of best fits for response distribution calculated using different information criteria implies the acceptance of an assumption of component behaviour under process variation in further IC analysis. Namely, best fit for circuit response distribution can be used as assumed and analyzed component's process variation distribution. However, the correct determination of the assumption requires considering parameters of random number generation included in MC analysis.

Further research on the subject can be oriented towards the justification of proposed assumption in the process variation-aware aging analysis [2], [3], [4]. In this way, the analysis of these two equally important factors of system and component performance would be significantly facilitated.

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