CMAC and Chebyshev Filters in Frequency Domain

Dragan Topisirović, Miona Andrejević Stošović, and Vančo Litovski

Abstract - In this paper we will compare the frequency domain behavior of the basic classes of polynomial low-pass filters with critical monotonic amplitude characteristics (CMAC), and Chebyshev filters. We will present phase and group delay characteristics of these filters, and also phase correctors' configurations that will be used to improve these characteristics, i.e. to reduce distortions of group delay. Conclusions as to which solution is preferable in a proper situation will be offered.

Keywords – CMAC filters, Chebyshev filter, frequency characteristics.

I. Introduction

In the available literature, there have not existed systematical comparisons of CMAC filters and their analogue with non-monotonic amplitude characteristic, known as Chebyshev filter [1]. If we want to compare them properly, we need to do that from several aspects: first, from the selectivity point of view; then, we can study their frequency domain characteristics, such as amplitude, phase, group delay, and finally, we can study their time domain characteristics. So, after these analyses, we expect to obtain answers to very interesting question: "Which characteristics are better, CMAC or Chebyshev?"

In this paper we will engage ourselves only in frequency characteristics of the basic classes of polynomial low-pass filters with critical monotonic amplitude characteristics, here referred to as CMAC, and Chebyshev filters.

Critical monotonic functions have the property that the amplitude characteristic in the pass-band has monotonic character with maximal number of inflection points. Also, it is need that amplitude characteristic has maximal number of inflexion points, with different abscissae. In that way, the first derivative of amplitude characteristic is equal to zero for maximum number of times, without changing its sign, meaning that its value is limited, and the sensitivity of the amplitude characteristic to changes of circuit parameters is reduced.

This is applied to four basic classes of CMAC filters [2]:

1. Maximally flat in the origin. This means all derivatives of $L_n(\omega^2)$ at the origin are to be zero.

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The class of filters thus obtained is called Butterworth`s after the author [2]. These will be here referred to as B-filters.

2. Maximum slope of the characteristic function at the edge of the pass-band. The class of filters so obtained is called L-filters and was introduced by Papoulis [4], [5]. The name L comes from the fact that in the original derivation Legendre polynomials were used. In some references [6] it is stated as "optimal filters" which is arbitrary.

3. Maximum asymptotic attenuation. This means the higher order coefficient in $L_n(\omega^2)$ has to be maximal. This class of filters was introduced by Halpern [7]. These will be here referred to as H-filters.

4. Least-squares-monotonic. In this case the returned power in the pass-band was minimized under the critical monotonicity criterion. This class was introduced by Raković and Litovski [8] and named LSM filters.

The paper is structured as follows: In the second chapter we will first give attenuation characteristics of both CMAC and Chebyshev filters, and then phase and group delay characteristics. In order to correct nonlinearities in the group delay characteristics, phase corrector structures are proposed in the third chapter. Finally, conclusions are given.

II. CMAC and Chebyshev Filter Characteristics

The main contribution of this paper is to present for the first time phase and group delay characteristics of CMAC filters.

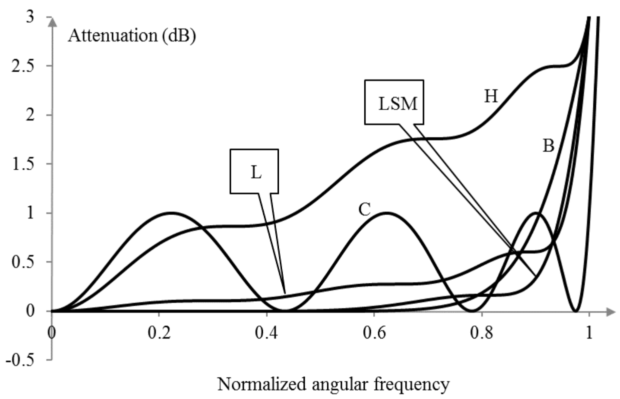


Figure 1. Attenuation characteristics of 7th order CMAC filters, and attenuation characteristics of Chebyshev filter with 1dB attenuation in the pass-band

In order to properly estimate these characteristics, we will also analyze Chebyshev filter with 1dB attenuation in the pass-band, what is average attenuation value of CMAC filters. We will consider here only 7th order filters.

If we analyse both filter types, aiming to compare these characteristics and to decide what filter to choose (or what approximation function to choose), we must consider several filter characteristics that participate in this choice, such as: time domain characteristics, sensitivity to the change of parameter values, what will not be considered in this paper, but in some of our future papers.

First, we will induce some definitions in order to present how frequency characteristics of the mentioned filters are obtained.

Attenuation characteristics of CMAC filters of 7th order, as well as Chebyshev characteristics are presented in Fig. 1. We can notice that Chebyshev filter, as it is defined, has 1dB attenuation at cut-off frequency, but other filters have 3dB attenuation.

However, in the pass-band, Chebyshev filter has larger attenuation at lower frequencies, what can be a disadvantage in the situations when the power density of the signal spectrum is larger at the lower frequencies (e. g. voice transmission). In such situations one needs to use either CMAC or Chebyshev filters with considerably lower attenuation in the pass-band, what causes a considerable reduction of selectivity, leading to increase of filter order for the sake of preserving selectivity.

Attenuation in the stop-band of the mentioned filters is presented in Figure 2. We can notice that from the selectivity point of view, Chebyshev filter dominates.

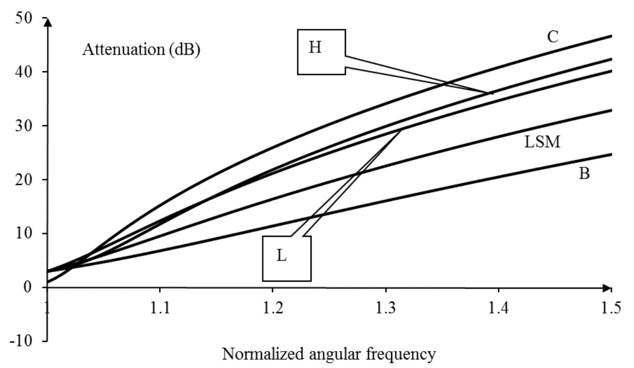


Figure 2. Attenuation characteristics of CMAC and Chebyshev filters in the stop-band

Phase characteristics of the above-mentioned filters are presented for the first time in Figure 3. It is known, but we can also see, that from the graphical presentation of the phase characteristic we can hardly conclude anything about phase distortions, so we usually use group delay characteristics.

They are presented in Figure 4. It is very noticeable here that Chebyshev filter exhibits large distortions of group delay, and also, that group delay is the least distorted by LSM and Butterworth filter.

III. Phase Correctors for LSM and Chebyshev Filters

By observing the group delay characteristics of the filters, we can deliver conclusions about phase corrector complexity that could be eventually used to correct phase delay characteristic, and also, about overall complexity of the cascade- filter+corrector.

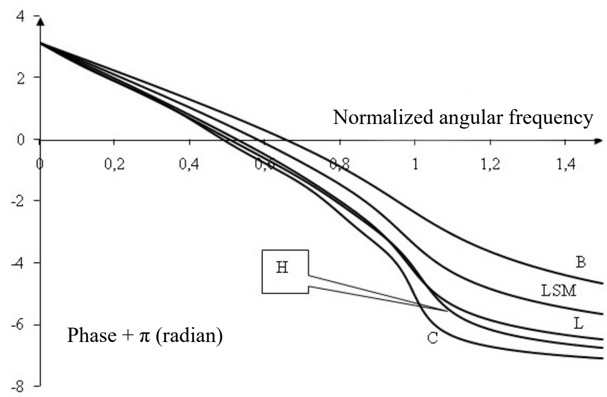


Figure 3. Phase characteristics of the 7th order CMAC and Chebyshev filters

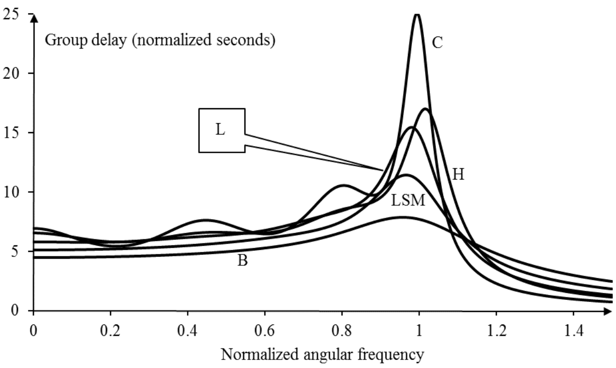
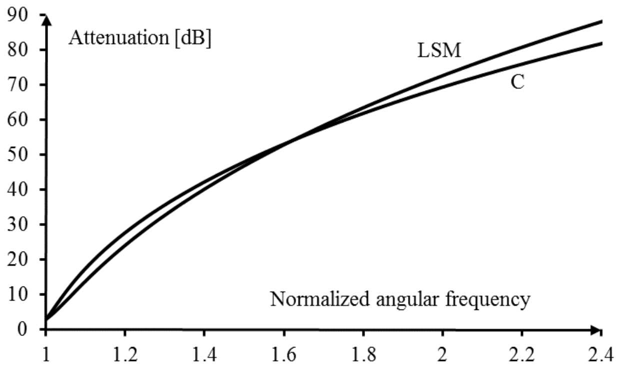


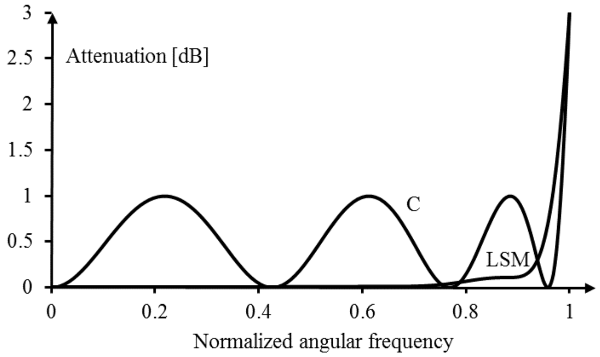
Figure 4. Group delay characteristics of CMAC and Chebyshev filters

As an example in this paper, we will use pair LSM-C (Chebyshev). From the filter theory, we can calculate the value of the highest coefficient in the square of the polynomial in the denominator of the amplitude characteristics of the 7th order Chebyshev filter with 1dB attenuation, and it is $(a_{2n}=a_{14})_{\text{Chebyshev}}=1060.56$. It represents the rate of asymptotic attenuation of the filter, i.e. its selectivity by Halpern criterion. Accordingly, asymptotic attenuation of this filter will be determined by $1060.56\cdot\omega^{14}$. LSM filter of the 9th order, whose highest order coefficient is $(a_{2n}=a_{18})_{\text{LSM}}=154.68$, has almost the same selectivity, so that asymptotic attenuation is determined by $154.68\cdot\omega^{18}$. Amplitude characteristics of these filters are shown in Figure 5, where we can see that we chose LSM filter with better attenuation characteristics in the pass-band, and almost the same in the stop-band.

If we realize these filters as passive ladder structures, this LSM filter would have two elements more (one inductor and one capacitor).



a)



b)

Figure 5. a) Attenuation in the stop-band and b) Attenuation in the pass-band of 7th order Chebyshev filter (1 dB, renormalized) and of 9th order LSM filter

If we design, for the two above-mentioned filters, phase correctors that should lead to small group delay error in greater part of the pass-band, considering the literature ([9], [10], [11]), we can expect that Chebyshev filter requires phase corrector whose order is for 2 higher than LSM filter corrector order (the optimum case-for Chebyshev). In the passive realization, considering complex zeroes, corrector cell has at least 5 elements, thereby using at least one transformer [12]. If we consider active cascade RC realization, extension of LSM filter would be achieved using 3rd order cell (Sallen-Key, for example), that realizes only zeroes in the infinity, but the extension of Chebyshev filter would require complex cell with noticeably greater number of elements (Tow-Thomas, for example).

As an illustration of this claim, we will consider complex filters obtained by cascade of the filter (CMAC or Chebyshev) and phase corrector. In order to obtain a fair comparison, we will use 9th order LSM filter and 7th order Chebyshev filter that is renormalized so that its amplitude characteristics achieves 3dB at cut-off frequency. This can be seen in Figure 1.

Comparison given in this paper is the first of that kind, and it could not be found in existing literature. This includes also the following examples.

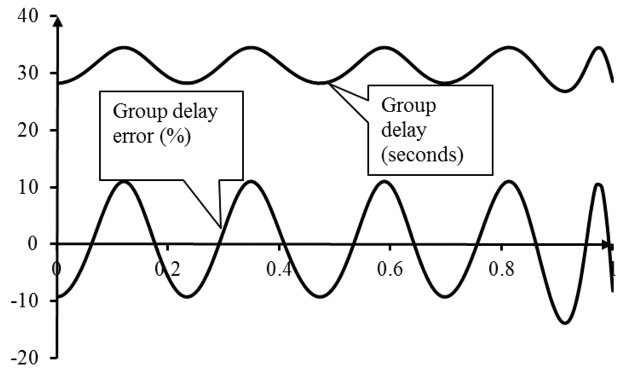


Figure 6. 7th order Chebyshev filter with 8th order corrector. Group delay (up) and error in approximation of constant group delay (down)

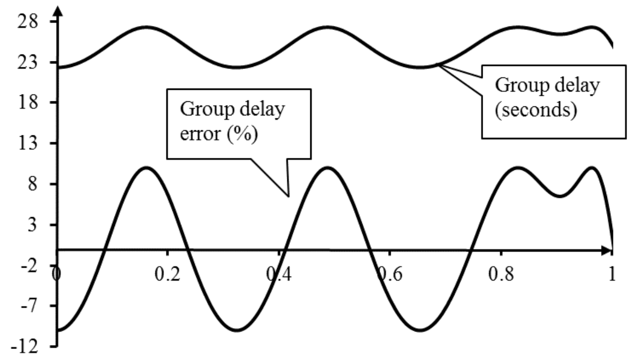


Figure 7. 9th order LSM filter with 6th order corrector. Group delay (up) and error in approximation of constant group delay (down)

If it is required that group delay is constant in the entire pass-band with 10% relative error, we obtain filters with characteristics presented in Figs. 6 and 7. Group delays and corresponding relative deviations are presented in these figures.

In Figure 6 these data are given for 7th order Chebyshev filter. In order to obtain approximation of constant group delay in the entire pass-band with 10% relative error, we needed 8th order corrector. According to theory, in ideal case, the value of group delay in pass-band should be: $\pi\cdot(n+2\cdot k)/2=\pi\cdot(7+2\cdot 8)/2=36$ s.

Here, n is order of the filter, k is order of the corrector. The value $(\tau_{d_mean})_{Chebyshev}=31.28$s, obtained by approximation is less because the part of the area under the group delay curve outspreads outside the pass-band. We should have in mind that with this corrector complexity, we cannot obtain group delay approximation in the entire pass-band, but this result is accepted as satisfactory.

Figure 7 represents group delay and relative deviations of the corrected 9th order LSM filter. In order to obtain approximation of constant group delay in the entire pass-band with 10% relative error, we needed 6th order corrector. This approximation is a bit better than the one in the Figure 6, but that is not of the big importance. Mean value of group delay of this combination is less than in the case of Chebyshev filter, and it is $(\tau_{d\_mean})_{LSM}$=24.77s. So, we conclude that the combination LSM filter+corrector would have smaller delay (in this case 21%) than the combination Chebyshev filter+corrector (that was reference when calculating relative deviation). Also, it is obvious that the condition that error of the group delay is smaller or equal 10% is easily fulfilled.

So, we can consider that in this way we obtained two filters with approximately same selectivity and approximately same group delay, but the corrected LSM filter exhibits less amplitude distortion in the pass band and smaller delay.

IV. CONCLUSION

In this paper we presented phase and group delay characteristics of CMAC filters for the first time. We used also Chebyshev filters frequency characteristics as a referent example for comparison.

We used pair the LSM- Chebyshev filter in the filter+corrector cascade to explore which combination would give better results, so we concluded that filters had almost the same selectivity, but the corrected LSM filter exhibited smaller delay and less amplitude distortion in the pass band.

ACKNOWLEDGEMENT

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