Small Signal Modeling of Series-Parallel Resonant Converters Based on Extended Describing Function

Milan Pajnić, Miroslav Lazić, Zoran Cvejić

Abstract - In this paper small signal modeling of series-parallel resonant converters based on extended describing function is presented. Resonant equivalent circuit state variables are approximated by their fundamental harmonics. By using extended describing function method and with separation of state variables, state space model of resonant converter is obtained. Small-signal model is analyzed in two main operation modes, when switching frequency is bellow and above frequency of resonant tank. Movement of zeros and poles of small signal model is presented in respect operation mode and ratio between switching and resonant frequency.

Keywords – Resonant converter, LLC converter, extended describing function, small-signal model.

I. Introduction

The resonant converters have advantages for high power or high-frequency power conversion. [1] Series or parallel resonant converters suffer from several drawbacks that limit its usefulness in many applications. Series resonant converters have small variation of dc conversion ratio over large range of switching frequency and are incapable of regulating output voltage when unloaded. In some applications, at light loads, the resonant current is reduced to a point where zero-voltage switching (ZVS) is lost.

These drawbacks are avoided with development of parallel, series-parallel, and many other higher-order resonant topologies [2]. The LLC series resonant converter (LLC- Series-parallel converter fig. 1) modifies the gain characteristics of a series resonant converter (SRC) and improves the light-load efficiency allowing boost mode operation.

Analysis methodologies that have been developed for modeling small-signal dynamics of power converters can be classified into two main categories. Averaging technique starts from state space description of each topology. It applies small ripple approximation and average time-invariant model is derived. After linearization and neglecting higher order terms, *s* domain transfer functions are obtained. *State space average* method has been used for converters with PWM regulation to analyze small signal dynamics, and provides accurate method for up to half switching frequency. The resonant converter switching frequency is close to resonant tank natural frequency, so states contains mainly switching frequency harmonics instead of low frequency content like in PWM converter. Since average method will eliminate the information of switching frequency, it cannot predict the dynamic performance of resonant converter.

For discrete modeling technique, state space system is also starting point. Equations are formed from equivalent circuit difference equations. However this method introduces error by approximation of transition in linear terms during one switching period.

The describing function concept has been introduced in [3]. The control-to-output and line-to-output describing functions are defined under the constant line voltage and constant control signal, respectively. Output control voltage is expressed as a Fourier series expansion. The resulting converter output voltage component at the same frequency as that of the perturbation signal is found by calculating the amplitude and phase of the fundamental term in the Fourier series. Extended describing function method proposed in [4] is a simplified modeling method based on description function method presented in [5]. Accuracy of this method has been verified in [6] based on analysis of series and parallel resonant converter. Detail small-signal characteristic of resonant LLC converter has been presented in [7] using simulation tool to emulate function of impedance analyzer to get the small signal response of resonant converter.

In Section II equivalent circuit of LLC converter is presented and extended describing function method is introduced. State space model is obtained with isolation of equation variables. Small-signal model characteristics of converter are presented in Section III. For a range of switching frequency two main modes have been identified and movement of zeros and poles presented. Section IV states the conclusions.

II. Small signal modeling

The circuit diagram of an LCC resonant converter is shown in Fig. 1, and equivalent circuit is shown in Fig. 2.

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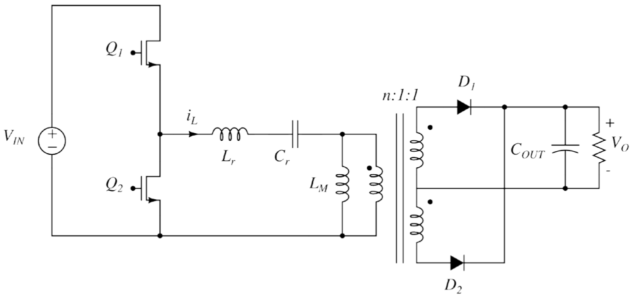

Fig. 1: Circuit diagram of an LCC resonant converter.

A. Nonlinear State Equation

The input voltage V_g is assumed to be symmetric square wave, with its magnitude proportional to the DC input voltage. The equivalent circuit provides the following nonlinear state space equations, where i_{L_r} , i_{L_M} and v_{C_r} are the state variables and V_O is the output variable:

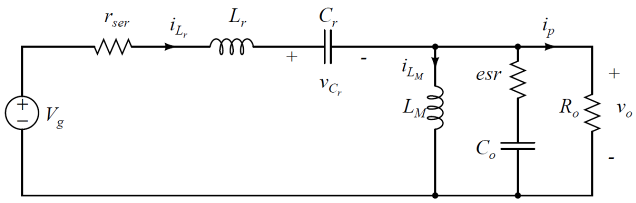

Fig. 2: Equivalent circuit diagram of LLC resonant converter

$$V_g = i_{L_r} r_{ser} + L_r \frac{di_{L_r}}{dt} + L_M \frac{di_{L_M}}{dt} + v_{C_r} \tag{1}$$

$$L_M \frac{di_{L_M}}{dt} = (i_{L_r} - i_{L_M}) R_O = \mathrm{sgn}(i_{L_r} - i_{L_M}) V_O \tag{2}$$

$$\frac{dv_{C_r}}{dt} = \frac{i_L}{C_r} \tag{3}$$

In this implementation of resonant converter, the output voltage is regulated by modulating the switching frequency where the operating point is determined by $\{v_g, \omega_s, R_O\}$.

B. Harmonic Approximation

According to [8] typical waveforms of the variables shown in Fig.2 can be approximated by fundamental harmonics, and the output capacitor voltage can be approximated by its dc component. Referring to [7], when converter is operating in continuous conduction mode with higher order of harmonics took into consideration the model will not be improved significantly, which is understandable since in continuous conduction mode LLC is operating like SRC. When converter is operating in discontinuous conduction mode observing fundamental component is not enough. With more harmonics considered, the model will be different especially in frequencies near double beat pole [7].

By making this assumption, we have:

$$i_{L_r}(t) = i_{L_{rS}}(t)\sin(\omega_s t) + i_{L_{rC}}(t)\cos(\omega_s t), \tag{4}$$

$$i_{L_M}(t) = i_{L_{MS}}(t)\sin(\omega_s t) + i_{L_{MC}}(t)\cos(\omega_s t), \tag{5}$$

$$v_{L_r}(t) = v_{L_{rS}}(t)\sin(\omega_s t) + v_{L_{rC}}(t)\cos(\omega_s t), \tag{6}$$

where terms $\{i_{L_{rS}}, i_{L_{rC}}, i_{L_{MS}}, i_{L_{MC}}, v_{C_{rS}}, v_{C_{rC}}\}$ are slowly time varying components.

C. Extended Describing Function

By using the extended describing function method [6], the nonlinear terms in (1-3) can be approximated either by the fundamental components terms or by the DC terms to give:

$$V_g \approx \frac{4}{\pi}\sin(\pi d) V_g \sin(\omega_s t) \tag{7}$$

$$\mathrm{sgn}(i_{L_r} - i_{L_M}) V_o = \frac{4}{\pi}\frac{i_{L_{rS}} - i_{L_{MS}}}{i_p} V_o \sin(\omega_s t) + \frac{4}{\pi}\frac{i_{L_{rC}} - i_{L_{MC}}}{i_p} V_o \cos(\omega_s t) \tag{8}$$

$$\left| i_{L_r} - i_{L_M} \right| = \frac{2}{\pi} i_p \tag{9}$$

$$i_p = \sqrt{(i_{L_{rS}} - i_{L_{MS}})^2 + (i_{L_{rC}} - i_{L_{MC}})^2} \tag{10}$$

With the small-signal modulation frequency lower than the switching frequency, by substituting (4-10) into (l-3), and by equating the coefficients of dc, sine, and cosine terms respectively, we get:

$$\frac{4}{\pi}\sin(\pi d) V_g = i_{L_{rS}} + L_r (di_{L_{rS}}/dt - \omega_S i_{L_{rC}}) + v_{C_{rS}} + L_M (di_{L_{MS}}/dt - \omega_S i_{L_{MC}}) \tag{11}$$

$$0 = i_{L_{rC}} + L_r (\frac{di_{L_{rC}}}{dt} - \omega_S i_{L_{rS}}) + v_{C_{rC}} + L_M (\frac{di_{L_{MC}}}{dt} - \omega_S i_{L_{MS}}) \tag{12}$$

$$L_M (\frac{di_{L_{MS}}}{dt} - \omega_S i_{L_{MC}}) = \frac{4}{\pi}\frac{i_{L_{rS}} - i_{L_{MS}}}{i_p} V_o \tag{13}$$

$$L_M (\frac{di_{L_{MC}}}{dt} - \omega_S i_{L_{MS}}) = \frac{4}{\pi}\frac{i_{L_{rC}} - i_{L_{MC}}}{i_p} V_o \tag{14}$$

$$C_r (\frac{dv_{C_{rS}}}{dt} - \omega_S v_{C_{rC}}) = i_{L_{rS}} \tag{15}$$

$$C_r (\frac{dv_{C_{rC}}}{dt} - \omega_S v_{C_{rS}}) = i_{L_{rC}} \tag{16}$$

$$V_o = \frac{2}{\pi} i_p R_o \tag{17}$$

$$\frac{dV_{oc}}{dt} = -\frac{V_{OC}}{C_O R_O} + \frac{2 i_p}{\pi C_O} \tag{18}$$

Equation (11-18) represents modulation equation, a nonlinear large-signal model of the LLC power stage. Inputs of (11-18) $\{v_g, \omega_s, d, i_p\}$ are varying slower than the switching frequency and by substituting (13) into (11) we get:

$$\frac{di_{L_{rS}}}{dt} = \omega i_{L_{rC}} - \frac{4}{\pi}\frac{V_o i_{L_{rC}}}{L_r i_p} - \frac{dv_{C_{rC}}}{L_r} + \frac{4}{\pi}\frac{\sin(\pi d)V_g}{L_r} - \frac{i_{L_{rS}}}{L_r} \tag{19}$$

For the small signal model to be presented in state space form, each state variable needs to be isolated in the equation and represented as the sum of the remaining variables. Since a ratio between state variables exists in (19) $i_{L_{rS}}/i_p$ where $i_p = \sqrt{(i_{L_{rS}} - i_{L_{MS}})^2 + (i_{L_{rC}} - i_{L_{MC}})^2}$ it is clear that this relationship of the state variables cannot be put into the states vector, therefore the ratio $i_{L_{rS}}/i_p$ is put into small signal form with the use of *Taylor Series* expansion. After expanding i_p, perturbing the large-signal model around the operating point $i_{L_{rS}} = I_{L_{rS}} + \tilde{i}_{L_{rS}}$ $i_{L_{rC}} = I_{L_{rC}} + \tilde{i}_{L_{rC}}$ $i_{L_{MS}} = I_{M_{rS}} + \tilde{i}_{L_{MS}}$ $i_{L_{MC}} = I_{M_{rC}} + \tilde{i}_{L_{MC}}$ and eliminating 2nd order small signal terms and DC constant terms, we get $\tilde{i}_{L_{rS}}/\tilde{i}_p$ in small signal form. Using previous algorithm same can be derived for $\tilde{i}_{L_{rC}}/\tilde{i}_p$, $\tilde{i}_{L_{MS}}/\tilde{i}_p$ and $\tilde{i}_{L_{MC}}/\tilde{i}_p$. After linearization around operating point and, model can be represented in state-space form as follows:

$$\frac{d\tilde{x}}{dt} = A\tilde{x} + B\tilde{u} \tag{20}$$

$$\tilde{y} = C\tilde{x} + D\tilde{u} \tag{21}$$

where:

$$x = \left[\tilde{i}_{L_{rS}}\ \tilde{i}_{L_{rC}}\ \tilde{i}_{L_{MS}}\ \tilde{i}_{L_{MC}}\ \tilde{v}_{L_{rS}}\ \tilde{v}_{L_{rC}}\ \tilde{v}_{CO}\right]^T \tag{22}$$

$$u = \left[\tilde{d}\ \tilde{\omega}\right]^T \tag{23}$$

$$y = \left[\tilde{v}_O\ \tilde{i}_p\right]^T \tag{24}$$

III. SMALL SIGNAL CHARACTERISTIC OF LLC RESONANT CONVERTER

Small signal characteristic of LLC converter (fig. 1.) is analyzed based on previous model. The simulation is performed for different operation modes in switching frequency range of 68-110 kHz. With use of extended describing function method, when converter is operating in continuous conduction mode, for $F \le 1$, where F is ratio between resonant and switching frequency, small signal characteristic is shown in figure 3. The graph contains one beat frequency double pole, one pole and one ESR left half plane zero. While operating near resonant frequency, beat frequency double pole will move to lower frequency. When switching frequency at resonant frequency the beat frequency double pole will split and become two real poles, figure 4. Moving to higher switching frequency will cause a double pole at lower frequency. Output capacitor ESR will cause a fixed frequency left half plane zero.

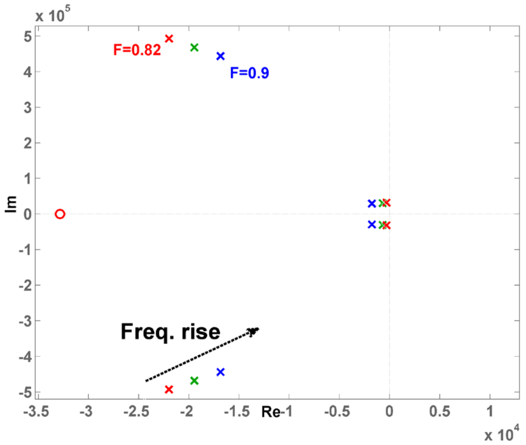

Fig. 3: Movement of poles and zeros for $F \le 1$ in CCM.

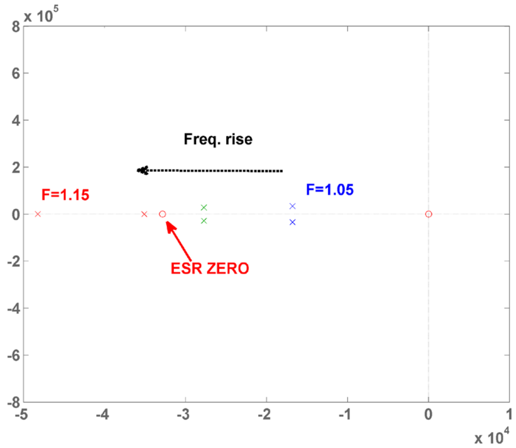

Fig. 4: Movement of poles and zeros for $F \approx 1$ in CCM.

When converter is operating in discontinuous mode, for $F \le 1$, small signal characteristic is shown in figure 5. In cases where switching frequency is lower than resonant frequency a right plane zero observable. In this case position of RHP zero is frequency dependent, but it doesn't shift to very low frequency even with F=0.8 or F=0.7.

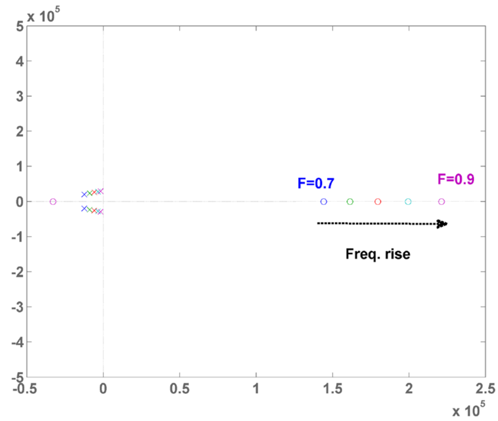

Fig. 5: Movement of poles and zeros for $F \le 1$ in DCM.

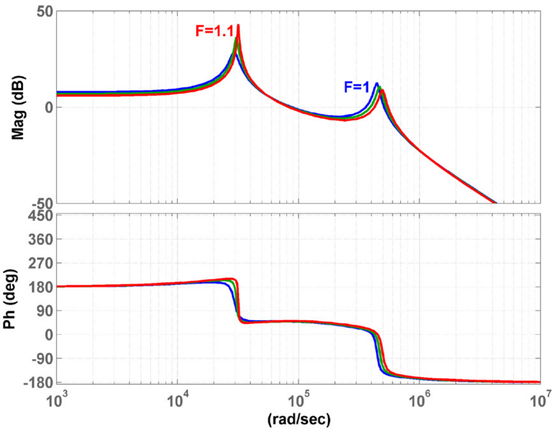

Fig. 5: Bode plot of control-to-output for CCM and $F \geq 1$.

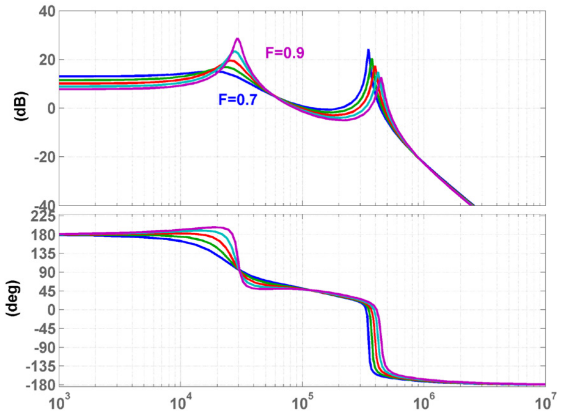

Fig. 6: Bode plot of control-to-output for DCM and $F \leq 1$.

In left half plane, referring to figure 5, there are two poles and one ESR zero. Compared to previous mode, poles they are less frequency dependent. Control to output transfer functions in different operation modes, CCM and DCM are presented in figure 5 and 6, respectively.

IV. Conclusion

In this paper a technique based on the extended-describing function was employed on modeling dynamics of LLC resonant converter. Resonant equivalent circuit state variables are approximated by their fundamental harmonics. To represent small signal model in state space form a separation of variables is performed with use of *Taylor Series* expansion. Movement of zeros and poles of small signal model is presented in respect operation mode and ratio between switching and resonant frequency. When resonant converter is operating in continuous conduction mode, the beat frequency double will move according to switching frequency and eventually split when switching and resonant frequency are equal. When resonant converter is operating in discontinuous conduction mode, all poles are less frequency dependent. Position of RHP zero is limited to high frequency and doesn't need special addressing in compensation design.

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